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Predictions of Heat Transfer and Pressure Drop for a Modified Power Law Fluid Flow in a Square Duct

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Abstract–Numerical Solutions for the Nusselt Numbers (CHF and CWT) and the Friction Factor times Reynolds Number have been obtained for fully developed laminar flow of a MPL (Modified Power Law) fluid within a square duct. The solutions are applicable to pseudoplastic fluids over a wide shear rate range from Newtonian at low shear rates through a transition region to power law behavior at higher shear rates. A shear rate parameter is identified, which allows the prediction of the shear rate range for a specified set of operating conditions. Numerical results of the Nusselt numbers (CHF and CWT) and the Friction factors times Reynolds number for the Newtonian and power law regions are compared with previous published results, showing agreement with 0.02% in Newtonian region and 4.0% in power law region.

Key words: Modified Power Law, Non-Newtonian Fluid, Pseudoplastic Fluid, Friction Factor, Shear Rate Parameter

INTRODUCTION

Because of wide applications in engineering, especially in the design of compact heat exchangers, much effort has been spent in determining the pressure drop and heat transfer characteristics of non-circular ducts. The prediction of pressure drop and heat transfer to fluids flowing in non-circular ducts are important in many engineering applications. Consequently, extensive analytical and experimental studies have been carried out on such flow systems. The analyses of the flows in non-circular ducts such as rectangular ducts are generally more complication than that of the circular pipe and the parallel plates. The investigation of the laminar flow and heat transfer behavior in a rectangular duct has become increasingly important as a result of the ongoing research of an advanced liquid cooling module for electronic packaging by a number of rectangular channels. Calculation of the friction factor for fully developed laminar flow in non-circular ducts requires a two dimensional analysis in contrast to the usual one dimensional analyses for a circular pipe or parallel plates. The boundary condition on the velocity for a fluid flowing through a non-circular ducts is the simple no-slip condition, the same as for circular pipe and parallel plates flows. For fully developed laminar flow of Newtonian and non-Newtonian power law fluids in a square duct, the solutions are well known for both the classical boundary conditions of constant wall temperature (CWT) and constant wall heat flux (CHF) and the pressure drop.

For Newtonian fluids, pressure drop and heat transfer coefficients were calculated by Shah and London [1978], Rothfus et al. [1964], Yang et al. [1998] etc. For power law fluids, Chandrupatla [1977], Wheeler and Wissler [1965], Kozicki and Tiu [1971], and Kozicki et al. [1966], Lee [1998] obtained those analytically and experimentally.

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An understanding of non-Newtonian fluid flow behavior will contribute substantially to the solution of a variety of ducts of arbitrary cross-section. It is of importance to have a knowledge of the characteristics of the pressure drop and the forced convection heat transfer in fully developed laminar non-Newtonian flow through a square duct to exercise an appropriate control over the performance of the heat exchanger and to economize the process. Furthermore, the results provide an appropriate basis for estimating the effects of the reduction of fluid frictional drag and heat transfer enhancement. Recently a large number of heat exchangers are designed and manu factured for the automotive and chemical process industries to heat or cool pseudoplastic fluids. Even today, there is a general lack of experimental data for heat transfer coefficients which are required for the heat exchanger designs. It is felt, however, that the rheological behavior can best be investigated with a well-defined geometry of ten found in industry, such as a square duct.

Non-Newtonian fluids usually have been assumed as power law fluids in the analysis. Many non-Newtonian fluids, however, have viscous properties which are different in the various shear rate ranges.

Although a power law model has been used extensively for calculating velocity profile and heat transfer coefficient in engineering, it has significant disadvantages that it only applies to the power law region in the flow curve and the apparent viscosity at the centroid of the duct becomes infinite.

A constitutive equation is one that relates the shear stress or apparent viscosity in a fluid to the shear rate through the rheological properties of the fluid. A convenient way to depict the constitutive equation is to plot a curve of apparent viscosity against shear rate. Fig. 1 shows such a graph which is indicative of the behavior of many purely viscous pseudoplastic fluids. In the lower shear rate range, the fluid is Newtonian and in the higher shear rate range the fluids acts as a power law fluid. Between these region is a transition range.

Such a rheological behavior in the transition zone causes several problems.

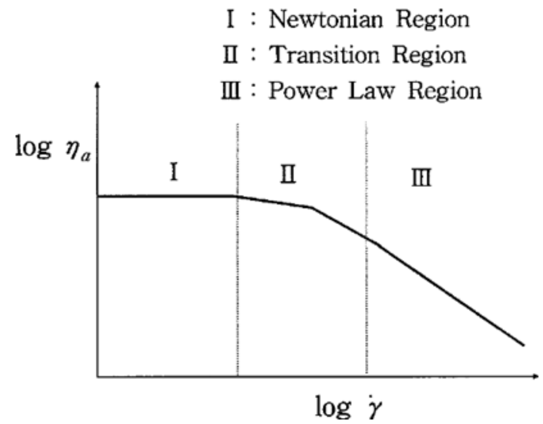

Fig. 1. Typical flow curve of pseudoplastic fluid.

1. It should be determined in which shear rate range the system is operating and if either of the Newtonian or power law solutions can be applied. This is not always simple because there is not a suitable shear rate parameter available and also the solutions were obtained independently. If the shear rate range falls within the transition zone then a "transition equation" must be applied for the type of non-Newtonian fluid considered here.

2. If the designer, as is often the case, builds a small prototype model, then the shear rate range as well as Reynolds number must be considered in the design of the larger system for similitude to be observed.

What is required to overcome these difficulties is a solution for a fluid which has rheological characteristics similar to Fig. 1.

A number of constitutive equations can describe the apparent viscosity-shear rate relation for fluids such as shown in Fig. 1. A convenient and useful equation of pseudoplastic fluid is the "Modified Power Law model" which was first proposed by Dunleavy and Middleman [1966].

$$\eta_a = \frac{\eta_0}{1 + \frac{\eta_0}{K}(\dot{\gamma})^{1-n}} \tag{1}$$

Inspection of Eq. (1) reveals that the apparent viscosity becomes equal to zero shear rate viscosity at very low shear rates and the fluid is operating in the Newtonian region of Fig. 1. At the higher shear rates the fluid becomes a power law fluid. At intermediate shear rates, there is a transition zone. An additional advantage of the modified power law model over other constitutive equations such as Sutterby [1966], Cross [1965], Carreau [1972], etc. is that the familiar Newtonian and power law Reynolds numbers are retained in the analysis.

The purpose of the present study is to extend our knowledge by presenting solutions for fluids having the rheological characteristics illustrated in Fig. 1 and to develop the relationships between the friction factor-Reynolds number and the heat transfer coefficients for a Modified Power Law fluid. Such a solution should have the characteristics that at low velocities (low shear rates) the Newtonian solution is an asymptote while at large shear rates the power law solution is an asymptote. In addition, the solution should predict the appropriate pressure drop and heat transfer behavior in the transition zone. Finally a parameter is needed to predict the shear rate range in terms of the operating characteristics of the system. For a circular tube [Brewster and Irvine, 1987], and concentric annulus [Capobianchi and Irvine, 1992], such solutions are available.

When using a particular constitutive equation, it is necessary to determine if the equation correctly describes the relation between the apparent viscosity and the shear rate for the particular fluid being considered. Thus it is required to measure the rheological properties in the constitutive equation and compare the equation of predictions with the experimental values of the apparent viscosity vs. the shear rate. This was done for the CMC (Sodium Carboxymethyl Cellulose) solutions by Park [1991, 1993].

ANALYSIS

The study of fully developed laminar flow in ducts comprises one of the fundamental and classical problems in fluid mechanics and heat transfer. Solutions to such problems are obtained by solving the appropriate forms of the momentum and energy equations along with the associated boundary conditions.

1. Pressure Drop

It is convenient to start with the conservation equations to solve a problem related to fluid flowing through duct. For steady flow of an incompressible fluid with negligible viscous dissipation, the governing equations depend on the apparent viscosity that related to the shear stress and shear rate.

For Newtonian fluids, the following simple relation $\tau_{ij}=\eta_a d_{ij}=\eta_0 d_{ij}$ has been used. But, for non-Newtonian fluids, the apparent viscosity is not a fluid property but is a function of velocity field. The momentum equation of non-Newtonian fluid depends on the relationship between the shear stress and the shear rate. For purely viscous non-Newtonian fluids, the following simple relation has been used [Hartnett and Kostic, 1989].

$$\tau_{ij} = \eta_a(\mathrm{I}, \mathrm{II}, \mathrm{III})\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

The apparent viscosity is a function of three invariants of the rate of deformation tensor d_{ij} for purely viscous non-Newtonian fluids. For an incompressible fluid, the first invariant vanishes and for a simple shear flow even the third invariant vanishes. The apparent viscosity is a function of the second invariant only as Aris [1962], Bird et al. [1977], and Wheeler and Wissler [1965].

$$\eta_a = \eta_a(\sqrt{\mathrm{II}/2}), \text{where } \mathrm{II} = 2\left\{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right\}$$

For power law fluids, the apparent viscosity can be represented as Wheeler and Wissler [1965].

$$\eta_a = K(\dot{\gamma})^{n-1} = K\left(\frac{\mathrm{II}}{2}\right)^{\frac{n-1}{2}}$$

The shear stresses which include gradients in both the y and z directions for a power law fluids are

$$\tau_{yx} = K\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}} \frac{\partial u}{\partial y}$$

$$\tau_{zx} = K\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}} \frac{\partial u}{\partial z}$$

From the power law relation $\tau=\eta_a\cdot\dot{\gamma}=K\dot{\gamma}^n$, the simple analytical models which neglect cross coordinate terms are

$$\tau_{yx}=K\left(\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y},\ \eta_{a,y}=K\left(\frac{\partial u}{\partial y}\right)^{n-1}$$

$$\tau_{zx}=K\left(\frac{\partial u}{\partial z}\right)^{n-1}\frac{\partial u}{\partial z}\ \eta_{a,z}=K\left(\frac{\partial u}{\partial z}\right)^{n-1}$$

The shear stresses include gradients both y and z directions for Modified Power Law fluids are as following.

$$\tau_{yx}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}}}\cdot\frac{\partial u}{\partial y}$$

$$\tau_{zx}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}}}\cdot\frac{\partial u}{\partial z}$$

Form the relation $\tau=\eta_a\dot{\gamma}$ and Eq. (1), the simple analytical models are as following.

$$\tau_{yx}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\partial u}{\partial y}\right)^{1-n}}\frac{\partial u}{\partial y}$$

$$\tau_{zx}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\partial u}{\partial z}\right)^{1-n}}\frac{\partial u}{\partial z}$$

For fully developed flow through ducts, it is possible to assume the following conditions;

$$\frac{\partial u}{\partial x}=0, v=w=0, p=p(x), u=u(y,z)$$

For a non-Newtonian Modified Power Law fluid flow through a square duct as shown in Fig. 2, the fully developed velocity field is described by the following momentum equation.

$$\frac{\partial}{\partial y}\left(\eta_{a,y}\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\eta_{a,z}\frac{\partial u}{\partial z}\right)=\frac{\partial p}{\partial x} \tag{2}$$

with boundary conditions

$$u(y,c)=0,\ \frac{\partial u(0,z)}{\partial y}=0$$

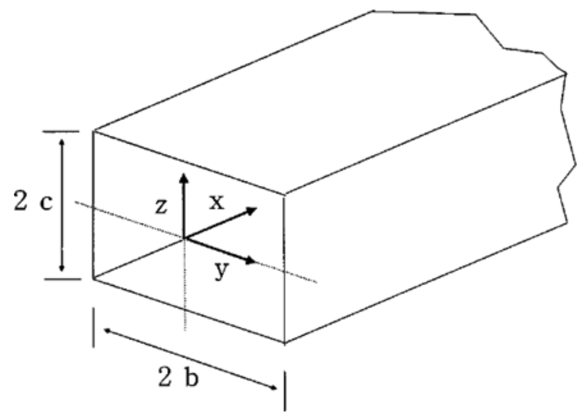

Fig. 2. Coordinate system for a rectangular duct.

$$u(b,z)=0,\ \frac{\partial u(y,0)}{\partial z}=0$$

The analytical models of the apparent viscosity for Modified Power Law fluids are as following.

$$\eta_{a,y}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\partial u}{\partial y}\right)^{1-n}},\ \eta_{a,z}=\frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\partial u}{\partial z}\right)^{1-n}}$$

Following dimensionless quantities may be defined

$$\alpha^*=\frac{c}{b},\ b^+=\frac{\alpha^*+1}{4\alpha^*},\ c^+=\frac{\alpha^*+1}{4}$$

$$y^+=\frac{y}{D_h},\ z^+=\frac{z}{D_h},\ f=-\frac{2D_h\frac{dp}{dx}}{\rho\bar{u}^2}$$

where the Darcy friction factor ($f=-8\tau_w/\rho\bar{u}^2$) is defined by a dimensionless pressure drop and D_h is hydraulic diameter ($D_h=4R_h=\{4\times$ cross−sectional area/wetted perimeter$\}=4bc/b+c$).

$$\eta_a^+=\frac{\eta_a}{\eta^*},\ u^+=\frac{u}{\bar{u}},\ \eta^*=\frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\bar{u}}{D_h}\right)^{1-n}}$$

$$Re_{D_h}=\frac{\rho\bar{u}D_h}{\eta_0},\ Re_g=\frac{\rho\bar{u}^{2-n}D_h^n}{K}$$

$$Re_m=\frac{\rho\bar{u}D_h}{\eta^*},\ \eta^*=\frac{\eta_0}{1+\beta}$$

$$\beta=\frac{Re_g}{Re_{D_h}}=\frac{\eta_0}{K}\left(\frac{\bar{u}}{D_h}\right)^{1-n},\ u^{++}=\frac{u^+}{\frac{1}{2}f\cdot Re_m}$$

$$Re_m=Re_{D_h}+Re_g=\frac{\rho\bar{u}D_h}{\eta_0}+\frac{\rho\bar{u}^{2-n}D_h^n}{K}=\frac{\rho\bar{u}D_h}{\eta_0}(1+\beta) \tag{3}$$

$$\eta_{a,y}^+=\frac{1+\beta}{1+\beta\left(\frac{1}{2}f\cdot Re_m\right)^{1-n}\left(\frac{du^{++}}{dy^+}\right)^{1-n}}$$

$$\eta_{a,z}^+=\frac{1+\beta}{1+\beta\left(\frac{1}{2}f\cdot Re_m\right)^{1-n}\left(\frac{du^{++}}{dz^+}\right)^{1-n}}$$

From Eqs. (1) and (4),
as $\beta\rightarrow 0$, $\eta_a\rightarrow\eta_0$ and $Re_m\rightarrow Re_{D_h}$
as $\beta\rightarrow$ very large, $\eta_a\rightarrow K(\dot{\gamma})^{n-1}$ and $Re_m\rightarrow Re_g$
For a non-Newtonian modified power law fluid through a square duct, the continuity equation can be expressed by the following equation

$$\bar{u}=\frac{1}{A_c}\int_{A_c}u\,dA_c=\frac{1}{bc}\int_0^c\int_0^b u\,dy\,dz \tag{4}$$

The dimensionless forms of Eqs. (2) and (4) are

$$f\cdot Re_m=\frac{(\alpha^*+1)^2}{8\alpha^*}\frac{1}{\int_0^{c^+}\int_0^{b^+}u^{++}dy^+dz^+} \tag{5}$$

$$\frac{\partial}{\partial y^+}\left(\eta_{a,y}^+\frac{\partial u^{++}}{\partial y^+}\right)+\frac{\partial}{\partial z^+}\left(\eta_{a,z}^+\frac{\partial u^{++}}{\partial z^+}\right)=-1 \tag{6}$$

with boundary conditions

$$u^{++}(y^+, c^+) = 0, \ \frac{\partial u^{++}(0, z^+)}{\partial y^+} = 0$$

$$u^{++}(b^+, z^+) = 0, \ \frac{\partial u^{++}(y^+, 0)}{\partial z^+} = 0$$

Thus Eq. (6) could give the complete solution for the fluids in Fig. 1 and the final results can be presented as the product of $f \cdot Re_m$ versus the shear rate parameter β.

2. Heat Transfer

When considering the fluid mechanics of non-Newtonian flow, the velocity boundary conditions at surfaces are quite straight forward. Except for certain classes of fluids which exhibit a slip phenomenon at solid boundaries, the boundary condition is normally taken as a no-slip or zero velocity at all solid surfaces. For heat transfer analyses however, the situation becomes more complicated. This is because there are many different ways to heat a well which in turn affects the type of thermal boundary conditions.

In general, the amount of heat transfer from a surface, or the temperature difference between the wall and the fluid are calculated using the equation

$$Q_{Total} = hA(T_W - T_f)$$

where:

Q_{Total}=heat transferred from the wall to the fluid [W]
h=convective heat transfer coefficient [W/m^{2}K]
A=heat transfer area [m^2]
$T_W - T_f$=temperature difference between wall and fluid [K]

Heat transfer coefficients are normally given in terms of Nusselt number (Nu=hL/k) where L is a characteristic length in a particular problem. Also, the fluid temperature, T_f, will depend upon a particular heat transfer situation. Both the characteristic length and the appropriate fluid temperature will be identified in the following presentations.

Since the heat transfer coefficient can vary considerably for different thermal boundary conditions especially for non-circular duct, it is important that the boundary conditions be specified correctly. Although the number of thermal boundary conditions is in principle infinite, several classical types have been identified and are in common use.

2-1. Energy Equation (CHF, H1)

Consider the case of constant heat flux (q_w) per unit area at wall in a square duct. Technically, constant heat flux problems occur in a plenty of situations: electric resistance heating, radiant heating, nuclear heating, and in counter flow heat exchangers.

The energy equation for the thermally developed flow in a square duct neglecting viscous dissipation and rate of energy generation [Incropera and DeWitt, 1996] with constant heat flux (CHF) can be written as

$$k\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho c_p u \frac{\partial T}{\partial x} \tag{7}$$

with boundary conditions

$$T(b, z) = T_W, \ \frac{\partial T(0, z)}{\partial y} = 0$$

$$T(y, c) = T_W, \ \frac{\partial T(y, 0)}{\partial z} = 0$$

The term "fully developed temperature profile" implies that there exists a generalized temperature profile that is invariant with duct length. The criterion for fully developed temperature profile can be expressed as

$$\frac{\partial}{\partial x}\left(\frac{T - T_\omega}{T_B - T_\omega}\right) = 0 \tag{8}$$

Writing the convection rate equation,

$$q_\omega = h(T_w - T_B) = \text{constant}$$

If h is a constant, then

$$T_\omega - T_B = \text{constant}$$

from which

$$\frac{dT_\omega}{dx} = \frac{dT_B}{dx}$$

Thus, from Eq. (7)

$$\frac{\partial T}{\partial x} = \frac{dT_\omega}{dx} = \frac{dT_B}{dx}$$

Substituting these into Eq. (8)

$$k\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho c_p u \frac{dT_B}{dx}$$

The following dimensionless quantities may now be defined

$$T^+ = \frac{T - T_W}{T_B - T_W}, \ T^{++} = \frac{T^+}{Nu_{H1}}$$

The dimensionless form of Eq. (7) becomes

$$\frac{\partial^2 T^{++}}{\partial y^{+2}} + \frac{\partial^2 T^{++}}{\partial z^{+2}} = -4u^+ \tag{9}$$

with boundary conditions

$$T^{++}(b^+, z^+) = 0, \ \frac{\partial T^{++}(0, z^+)}{\partial y^+} = 0$$

$$T^{++}(y^+, c^+) = T_W, \ \frac{\partial T^{++}(y^+, 0)}{\partial z^+} = 0$$

Considering the definition of bulk temperature, T_B:

$$T_B = \frac{\int_{A_c} uT dA_c}{A_c u} \tag{10}$$

For the square duct geometry, Eq. (10) may be rewritten in dimensionless from

$$1 = \frac{(\alpha^* + 1)^2}{16\alpha^*} \frac{1}{\int_0^{c^+}\int_0^{b^+} u^+ T^+ dy^+ dz^+} \tag{11}$$

Introducing the definition of T^{++} and solving for the Nusselt number gives

$$Nu_{H1} = \frac{(\alpha^* + 1)^2}{16\alpha^*} \frac{1}{\int_0^{c^+}\int_0^{b^+} u^+ T^{++} dy^+ dz^+} \tag{12}$$

2-2. Energy Equation (CWT, T)

Next consider the case where the surface temperature (T_ω) is constant. This is another very common convection application, which occurs in such heat exchangers as evaporators, condensers.

The energy equation for the thermally developed flow in a square duct neglecting viscous dissipation and rate of energy generation [Incropera and DeWitt, 1996] with constant wall temperature (CWT) can be written as

$$k\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c_p u\frac{\partial T}{\partial x} \tag{13}$$

with boundary conditions

$$T(b,z)=T_w,\ \frac{\partial T(0,z)}{\partial y}=0$$

$$T(y,c)=T_w,\ \frac{\partial T(y,0)}{\partial z}=0$$

For constant wall temperature (T_ω=constant)

$$\frac{dT_\omega}{dx}=0$$

and Eq. (7) reduces to

$$\frac{dT}{dx}=\frac{T_\omega-T}{T_\omega-T_B}\frac{dT_B}{dx}$$

substituting in Eq. (13),

$$k\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C_p u\frac{T_\omega-T}{T_\omega-T_B}\frac{dT_B}{dx}$$

Defining the following dimensionless quantities.

$$T^+=\frac{T-T_\omega}{T_B-T_\omega}$$

The dimensionless form of Eq. (13) becomes

$$\frac{\partial^2 T^+}{\partial y^{+2}}+\frac{\partial^2 T^+}{\partial z^{+2}}=-4u^+T^+Nu_T \tag{14}$$

with boundary conditions

$$T^+(b^+,z^+)=0,\ \frac{\partial T^+(0,z^+)}{\partial y^+}=0$$

$$T^+(y^+,c^+)=0,\ \frac{\partial T^+(y^+,0)}{\partial z^+}=0$$

Eqs. (12) and (14) were solved numerically to obtain the relationship of Nusselt number vs. the shear rate parameter β for constant heat flux and constant wall temperature with the dimensionless velocity distribution, u^+ calculated from the solution of the previous momentum equation.

NUMERICAL ANALYSIS

The numerical formulation and solution were relatively straightforward. An Alternating Direction Implicit method was used with successive overrelaxation. The algorithm was as follows:

Step 1 : Specify values of n, α^* and β.
n=1.0, 0.9, 0.8, 0.7, 0.6, 0.5, 0.4
α^*=1.0 (for square duct)
$10^{-4}\le\beta\le10^4$

Step 2 : Assume a velocity profile starting with a $u^{++}(y^+, z^+)=0$ for a Newtonian fluid. The Newtonian velocity profile may then be used as the initial velocity profile for the non-Newtonian MPL calculation.

Step 3 : Calculate $\eta^+_{a,y}$, $\eta^+_{a,z}$ fields by using the assumed velocity field.

Step 4 : Solve for $u^{++}=u^{++}(y^+, z^+)$ by using ADI (Alternating Direction Implicit) method and obtain fRe_m by Simpson's rule. TDMA (Tri-Diagonal Matrix Algorithm) may be used for obtaining the velocity profile.

Step 5 : Calculate new $\eta^+_{a,y}$, $\eta^+_{a,z}$ from the new value of the velocity field.

Step 6 : Calculate a new $u^{++}(y^+, z^+)$ and $f\cdot Re_m$.

Step 7 : Compare the $f\cdot Re_m$ value with the value calculated in step 4.

Step 8 : Use the new $f\cdot Re_m$ to calculate a new u^{++} and $f\cdot Re_m$ until convergence.

Step 9 : Obtain the u^{++} field and $f\cdot Re_m$.

Step 10 : Use the u^{++} field and $f\cdot Re_m$ to obtain temperature profile by TDMA.

Step 11 : Use the u^{++}, $f\cdot Re_m$, and T^{++} to calculate Nusselt number by Simpson's rule.

RESULTS AND DISCUSSION

A number of modified power law numerical solutions have been obtained, which for fully developed laminar duct flows include friction factors and Nusselt numbers for a square duct. In the following, the results of these analyses will be presented in graphical form. These results are shown in Fig. 3 to Fig. 5.

1. Friction Factors for Fully Developed Flows

A numerical solution to Eq. (5) for a square duct are shown in Fig. 3. The figure illustrates that in a quantitative sense, β defines the three regions as follows.

Region I - Newtonian $\beta<10^{-3}$

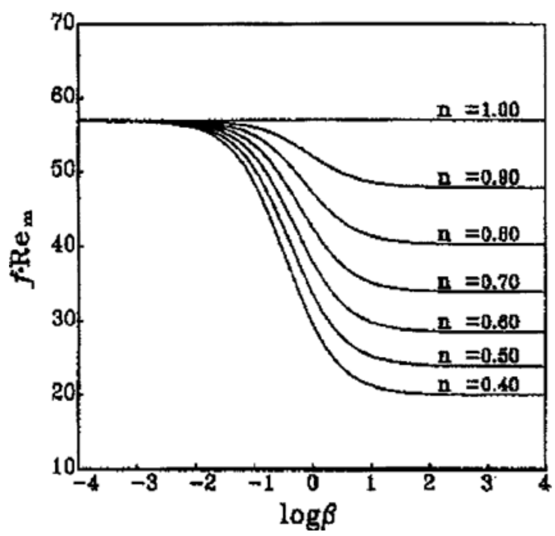

Fig. 3. $f\cdot Re_m$ for a MPL fluid in a square duct.

Region II - Transition $10^{-3}<\beta<10^{3}$
Region III - Power Law $\beta>10^{3}$

Fig. 3 also illustrates several important features of modified power law system. First, for complete similarity modeling, the modified Reynolds number Re_m and the parameter β must both be considered. Also, a considerable difference exists if it is assumed that the system is operating in region III when it actually is operating in Region I. Simple calculations show that errors in pressure drop predictions can be as large as several hundred percent if such an uncertainty exists in correct operating region.

As the shear rate parameter increases, the Reynolds number increases. As the power law flow index(n) increases, the tendency increases to retain Newtonian characteristics at low Reynolds numbers. As the flow index decreases, the tendency increases to retain the characteristics of power law fluid at high Reynolds numbers.

The numerical results of the friction factor and Reynolds numbers relations and the Nusselt numbers for the Newtonian and the power law region were compared with other previously published asymptotic results [Shah and London, 1978; Rothfus et al., 1964; Chandrupatla, 1977; Wheeler and Wissler, 1965; Kozicki and Tiu, 1971; Kozicki et al., 1966]. For Newtonian fluid flow through a square duct, the differences of the friction factors times the Reynolds numbers between the results of Shah and London [1978] and the present results are less than 0.02%.

2. Fully Developed Laminar Heat Transfer

For Newtonian fluid flow through a square duct, the differences of the Nusselt number (CHF and CWT) between of the results of Shah and London [1978] and the present results are less than 0.02%. These results are shown in Table 1.

For power law fluids which various flow indices (n=0.4, 0.5, ⋯, 1.0) the differences of the friction factors times the generalized Reynolds numbers between the results of Kozicki et al. [1966] and the present results with $\beta=10^{4}$ are less than 0.9%. The differences of the friction factors times the generalized Reynolds numbers between the results of Wheeler and Wissler [1965] and the present results with $\beta=10^{4}$ are less than 4.0%. These results are shown in Table 2.

Table 1. Comparison of $f\cdot Re_{D_h}$, Nu_{H1}, and Nu_T of Newtonian fluid

	$f\cdot Re_{D_h}$	Nu_{H1}	Nu_T
(1)	56.9083	3.6079	2.9760
(2)	56.9184	3.6070	2.9760

(1) Shah and London [1978].
(2) Present calculation

Table 2. Comparison of $f\cdot Re_g$ of power law fluids

n	(1)	(2)	(3)
1.0	56.912	56.876	56.910
0.9	47.640	47.620	47.887
0.8	39.692	40.244	40.293
0.7	33.080	33.804	33.894
0.6	27.540	28.356	28.489
0.5	22.932	23.740	23.909
0.4	-	19.816	20.008

(1) Wheeler and Wissler [1965].
(2) Kozicki et al. [1966].
(3) Present calculation

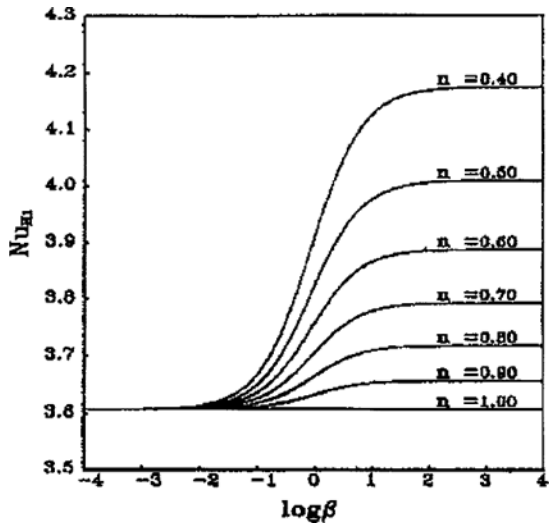

Fig. 4. Nu_{H1} for a MPL fluid in a square duct.

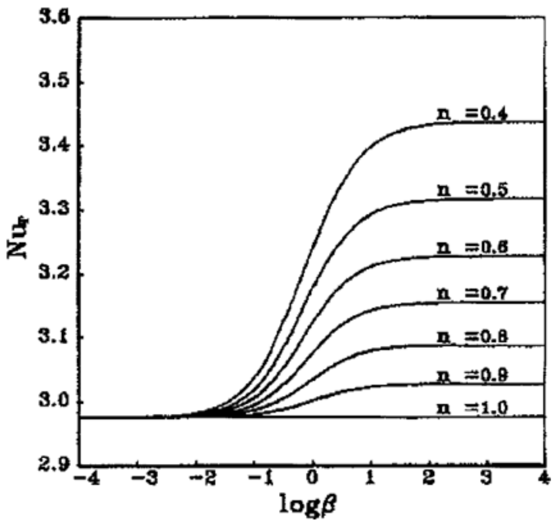

Fig. 5. Nu_T for a MPL fluid in a square duct.

Figs. 4 and 5 show the fully developed Nusselt numbers versus the shear rate parameter for a square duct for the thermal boundary conditions of constant wall temperature (CWT) and constant heat flux (CHF). It is interesting to note that the effect of the shear rate parameter β is much less for the fully developed Nusselt numbers than for the product $f\cdot Re_m$. Thus it would appear that the effect of β on the hydrodynamic design is much more critical for the thermal design.

For power law fluid flow through a square duct, the differences of the Nusselt numbers (CHF and CWT) between the results of the results of Chandrupatura [1977] and the present results are less than 4.0%. These results are shown in Table 3.

The shear rate parameter defines the transition region (approximately $10^{-2.5}\leq\beta\leq10^{2.5}$) and is useful to estimating whether the fluid is a fully developed Newtonian fluid ($\beta\leq10^{-2.5}$) or a fully developed Power Law fluid ($\beta\geq10^{2.5}$). Thus the shear rate parameter β can be used to determine in which of the three regions (Fig. 1) a particular

Table 3. Comparison of Nu of power law fluids

n	Nu_{H1}(1)	Nu_T(1)	Nu_{H1}(2)	Nu_T(2)
1.0	3.612	3.607	2.975	2.976
0.9	3.648	3.657	2.997	3.027
0.8	3.689	3.718	3.030	3.087
0.8	3.741	3.793	3.070	3.155
0.6	3.804	3.887	3.120	3.227
0.5	3.889	4.010	3.184	3.317
0.4	-	4.175	-	3.436

(1) Chandrupatla [1977].
(2) Present calculation

system is operating.

CONCLUSION

Fluids which are called "power law" sometimes follow that constitutive equation, but depending upon operating shear rate they can also act as Newtonian or Transitional fluids.

By using a more general constitutive equation, the modified power law equation, solutions are possible which take this shear rate dependence into account and through a dimensionless shear rate parameter enable an appropriate choice of the pressure drop and heat transfer solutions.

This situation has been examined for forced laminar convection in ducts and it is illustrated that serious errors can results if the incorrect shear rate solution is used. Of particular concern are duct flows operating at low Reynolds numbers.

Numerical solutions for laminar fully developed flow were obtained for friction factor times Reynolds number for MPL fluid flow through square duct. By using the MPL constitutive equation, we obtained solutions which are applicable over a wide shear rate range of pseudoplastic fluids from Newtonian behavior to the higher shear rate range. A shear rate parameter was identified which specifies whether a particular system for a typical pseudoplastic fluid is operating in the Newtonian, transition, or power law region. The numerical results of the pressure drop and heat transfer augmentation for the Newtonian and power law regions were compared with other previously published asymptotic results as discussed earlier.

As the shear rate increases, the tendency increases to retain power law fluid characteristics at high Reynolds numbers. As the shear rate decreases, the tendency increases to retain Newtonian fluid characteristics at low Reynolds numbers.

During the analysis, the shear rate parameter β can be used to determine that the particular system is operating in one of the three regions (Figs. 1 and 3).

For pseudoplastic non-Newtonian fluid, the Modified Power Law model is recommended to use because the fluid properties have big discrepancies between the power law model and the actual values in low and medium range of shear rates.

The numerical solution makes possible the conservation of similitude when designing duct systems for such fluids as modified power law fluids since both the appropriate Reynolds number and the shear rate ranges are considered.

From a comparison of the numerical calculations between Newtonian and non-Newtonian fluid flow it is obvious that for the thermal boundary conditions (CHF and CWT) a non-Newtonian fluid with flow behavior index less than one gives a higher heat transfer coefficient than a Newtonian fluid. Due to the reduction in friction power requirement and the augmentation in heat transfer rates, modified power law fluids seem to be better working fluids in heat exchanger compared to Newtonian fluids. On the other hand, the use of appropriate modified power law fluids may lead to heat transfer enhancement without the handling difficulties.

The feasibility of application of this friction factor and Reynolds number relation will be valid for the determination of cross-sectional shapes and tortuosities of creviced channels in packed beds and porous media; and the heat transfer augmentation for modified power law fluids in a square duct can be applied for the design of a liquid cooling module in electronic packaging, where uneven thermal boundary conditions with non-circular ducts are commonly employed.

NOMENCLATURE

A	: heat transfer area [m^2]
A_c	: cross-sectional area of duct [m^2]
b	: one half of duct width [m]
b^+	: dimensionless duct width [-]
c	: one half of duct height [m]
c^+	: dimensionless duct height [-]
C_p	: specific heat [J/kg·K]
D_h	: hydraulic diameter [4×cross-sectional area/wetted perimeter= 4bc/(b+c)] [m]
d_{ij}	: shear rate tensor [1/s]
f	: Darcy friction factor [$-2(dp/dx)D_h/\rho\bar{u}$] [-]
h	: convective heat transfer coefficient [$W/m^2 \cdot K$]
K	: power law consistency [Ns^n/m^2]
k	: thermal conductivity [W/m·K]
Nu	: Nusselt number [-]
Nu_{H1}	: Nusselt number of CHF [-]
Nu_T	: Nusselt number of CWT [-]
n	: power law flow index [-]
Q_{Total}	: heat transferred from the will to the fluid [W]
$\dot{q}_W$	: heat flux at wall [$J/s \cdot m^2$]
R_h	: hydraulic radius (cross-sectional area/wetted perimeter= bc/(b+c)) [m]
Re_{D_h}	: Newtonian Reynolds number ($\rho\bar{u}D_h/\eta_0$) [-]
Re_g	: power law Reynolds number ($\rho\bar{u}^{2-n}D_h^n/K$) [-]
Re_m	: modified power law Reynolds number ($\rho\bar{u}D_h/\eta^*$) [-]
T	: temperature [K]
T^+	: dimensionless temperature [-]
T^{++}	: dimensionless temperature [-]
T_B	: bulk temperature [K]
T_f	: fluid temperature [K]
T_W	: wall temperature [K]
u	: velocity in flow direction [m/s]
$\bar{u}$	: mean velocity in flow direction [m/s]
u^+	: dimensionless velocity in x-direction ($u/\bar{u}$) [-]
u^{++}	: dimensionless velocity in x-direction ($2u^+/f \cdot Re_m$) [-]
x, y, z	: coordinates

Greek Letters

α^* : aspect ratio (c/b) [-]
β : shear rate parameter $[(\eta_0/K)(\bar{u}/D_h)^{1-n}]$ [-]
$\dot{\gamma}$: shear rate [1/s]
η_a : apparent viscosity $(\tau/\dot{\gamma})$ [Ns/m^2]
η_0 : zero shear rate viscosity [Ns/m^2]
η^* : reference viscosity $(\eta_0/(1+\beta))$ [Ns/m^2]
η^+ : dimensionless viscosity (η_a/η^*) [-]
ρ : fluid density [kg/m^3]
τ : shear stress [Ns/m^2]
I, II, III : invariants of shear rate tensor [-]

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